On Crop Certainty

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Introduction

Much more attention has been paid during the last few years to determining with what degree of certainty a particular yield level be maintained from year to year. From the point of view of the national economy and the individual farmer, it would be of great benefit if large and high quality yields could be continuously obtained from field crops with as high a degree of certainty as possible.

The certainty of obtaining a crop (here called more briefly *crop certainty*) made an appearance in connection with the so-called green revolution. It had been possible in the environmental conditions prevailing in the developing countries to develop varieties of rice and wheat that were more productive than the types used earlier. Agriculture based on such premises has however turned out to be rather vulnerable. During unfavourable growing seasons and at times when the numbers of plant pests were higher than normal, the crops have suffered considerable damage. The crop certainty of the new varieties was lower than that of the varieties used earlier. The term ecological tolerance is also used to describe this phenomenon (Wricke, 1965). In the development of new varieties of rice and wheat, some of the original ecological tolerance had been lost.

Crop certainty is also a topic of current importance in Finland. As a direct result of plant breeding and the development of cultivation techniques, yields per hectare have increased. Are there any differences in the present crop certainty between, on the one hand, different cultivated plants, and on the other the crop certainty, prevailing in different parts of the country? Are there likely to be any serious repercussions in present day cultivation if during some year the weather conditions or numbers of pests are particularly exceptional?

Crop certainty can be quantified, for instance, by means of the coefficient of variation (cf. Paatela & Suomela, 1960). It thus becomes possible to make comparisons between different crops because standard deviations of the yield are measured as a percentage of the mean values. The necessary calculations are simple and quick to carry out, but there is a disadvantage concerning the dependance on the mean. As the yield level increases the value for the coefficient of variation decreases, despite the fact that the size of the yield variations remains constant. Thus although the crop certainty improves, the increase is only seeming.

Calculation of crop certainty based on probability is presented in this study. The study is a theoretical background for the crop certainty of different crops in a project being carried out in different parts of Finland. Answers to the following questions are sought:

1. What is the crop certainty that the size of the yield will exceed a certain yield level?
2. What is the crop certainty that the quality of the yield will exceed certain requirements for quality?
3. What is the crop certainty that the size of the yield will exceed a certain yield level, and that simultaneously the quality of the yield will exceed certain requirements for quality?

Material and Tests of hypotheses

Sugar beet (*Beta vulgaris* L.) was chosen as a test plant because the most accurate long term yield data is available for this crop. Apart from the size of the yield, the sugar content which can be used as a measure of the quality, has also been determined for each year. The crop certainty will be calculated for the areas where sugar beet is grown under contract for two sugar extraction factories situated in south-western Finland, Salon Sokeritehdas Co. and Juurikassokeri Co. (in Naantali) (Fig. 1). The average annual yield and sugar content for both areas during the

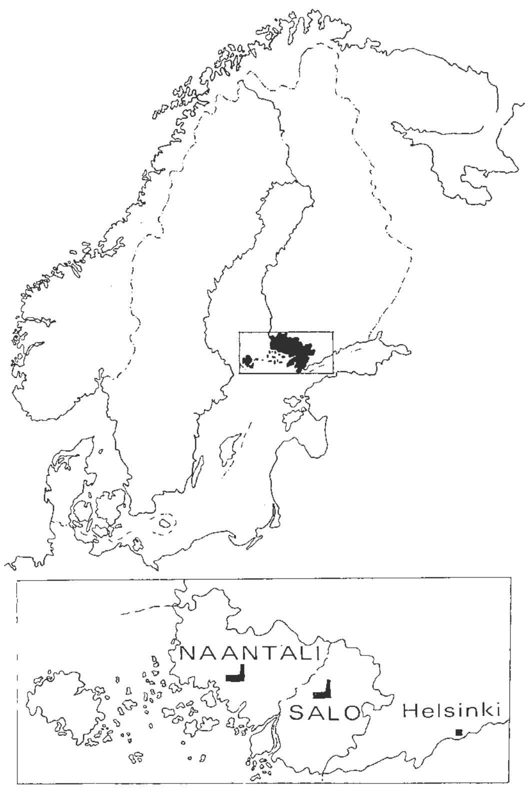


Fig. 1. The sugar beet areas of Salon Sokeritehdas Co and Juurikassokeri Co (in Naantali).

period 1953–1974 has been used as the primary data.

The yield levels in both Naantali and Salo areas have clearly risen since 1953. The linear trend in the yields can be put in the mathematical form

$$y_S = 18\,800 + 558\,(z - 1953), \quad (1)$$

$$y_N = 19\,800 + 641(z - 1953), \quad (2)$$

where y_S and y_N are the annual sugar beet yields (kg/ha) for the Salo and Naantali areas, and z is the year.

The increasing trend of the yield level is statistically significant, since the t-values of the slope of the trend lines for both the Salo area ($t_S = 2.7$) and the Naantali area ($t_N = 2.9$) are greater than the critical value $t_{20}(0.05) = 2.086$. Therefore, according to this trend, the sugar beet yield for 1975 in the Salo area would be 31 100 kg/ha and in the Naantali area 33 900 kg/ha.

The sugar content shows neither an increasing or decreasing trend, since the t-values of both

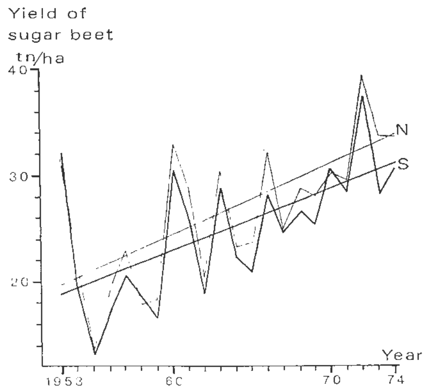


Fig. 2. The average annual yields of sugar beet for the Salo (S) area and the Naantali (N) area and the estimated trend lines.

slopes, $t_S = -0.2$ and $t_N = -1.2$ are in absolute terms smaller than the critical value. Thus on average, the sugar content in both areas has remained unchanged over the last 22 years although the annual variations have been large. The sugar content of sugar beet growing in the Naantali area has been slightly higher than in the Salo area. The mean sugar contents are $\bar{x}_S = 15.2\%$ and $\bar{x}_N = 15.5\%$.

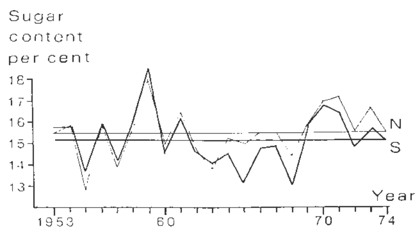
Next we have to determine whether or not the annual yields and sugar contents of the material are normally distributed. In order to illustrate the problem, the sugar content is treated first because it is not necessary to take any changing trend into account. The standard deviation (s_x) of the sugar content is obtained using the formula

$$s_x = \sqrt{\sum_{i=1}^{n} (x_i - \bar{x})^2/(n-1)}. \quad (3)$$

The deviation for the Salo area is $s_{xS} = 1.25\%$ and for the Naantali area $s_{xN} = 1.19\%$.

Let us make the hypothesis that the annual sugar content (x_S) of the Salo area has a normal distribution where the expected value for the

Fig. 3. The average annual sugar contents and corresponding mean values.



sugar content (15.2%) and the variance ((1.25%)²) are the parameters, i.e.

$$x_S \sim N(15.2, 1.25^2). \tag{4}$$

The hypothesis is tested by the Kolmogorov–Smirnov test (see e.g. Lindgren, 1970, p. 329), because the number of yearly observations (22) is so small that the χ^2 test cannot be used. The sample distribution function (Fig. 4) is used in the test. The function indicates in how many years has the sugar content been below a certain level. In the Salo area it has fallen below 13.5% once during ten years, below 15.5% every other year and below 16.5% in nine years out of ten.

The cumulative distribution function calculated according to hypothesis (4) has also been drawn in Fig. 4. The Kolmogorov–Smirnov test is used to find the largest distance between sample distribution function and the normal cumulative distribution function. This occurs at a point which corresponds to a sugar content of 14.8%, where the difference between the observed and calculated values is $D = 0.118$. This is smaller than the critical value $D_{22}(0.20) = 0.220$ (Lindgren, 1970, p. 486). Thus, the hypothesis that the sugar content for the Salo area has a normal distribution can be accepted. The sugar content for the Naantali area can be studied in a similar manner. It also has a normal distribution, because the statistic obtained, $D = 0.136$, is below the critical value.

When the size of the sugar beet yield is examined, the effect of the increasing trend must be taken into account. In this case we have to study whether or not the yearly deviations (u_i) from the trend line are normally distributed. The standard error (s_y) is calculated using the formula

$$s_y = \sqrt{\sum_{i=1}^{n} u_i^2/(n-2)}. \tag{5}$$

The standard error in the Salo area is $s_{yS} = 5\ 090$ kg/ha and in the Naantali area $s_{yN} = 5\ 120$ kg/ha.

Let us make the hypothesis that the size of the annual sugar beet yield (y_S) in the Salo area has a normal distribution where the trend yield for the year in question and the variance ((5 090 kg/ha)²) are the parameters. For instance, in 1975

$$y_S \sim N(31\ 100, 5\ 090^2). \tag{6}$$

The hypothesis can be tested in the same way as earlier using Kolmogorov–Smirnov test. A test statistic is now $D = 0.136$. The corresponding

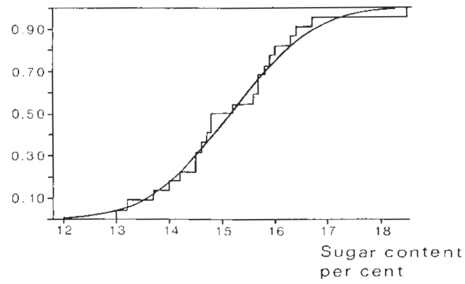


Fig. 4. The sample distribution function (step line) and the normal cumulative distribution function concerning sugar content in the Salo area.

value for the Naantali area is $D = 0.149$. These values are both smaller than the critical value (0.220). Thus we can accept the hypothesis, that the annual deviations in the size of the sugar beet yield from the trend line are normally distributed.

Whether or not the sugar content and the size of the sugar beet yield are dependent on each other still needs to be investigated. Since the hypotheses that these variables have normal distribution could be accepted, it is sufficient to study whether there is any correlation between the sugar content and the size of the sugar beet yield. The correlation for the Salo area is $r_S = 0.0095$ and for the Naantali area $r_N = 0.0062$. Both of these values can be considered to be almost equal to zero. Therefore it can be stated that during the period under study (1953–1974), the sugar content and the size of the sugar beet yield have not been dependent on each other.

Crop Certainty

Crop certainty is defined as the probability, calculated from the normal distribution, when a certain quantity or quality level is required. Let us take as an example the size of the sugar beet yields for the Salo area. The sugar beet yield for 1975 should be at least 35 000 kg/ha. What is the crop certainty of obtaining such a yield?

It was earlier demonstrated that the size of the sugar beet yields in the Salo area have a normal distribution and the parameters for 1975 are the trend yield 31 100 kg/ha and the variance (5 090 kg/ha)² (Fig. 5). The crop certainty is obtained by calculating what percentage of the total area under the curve in Fig. 5 is taken up by the shaded area. The crop certainty in this case is about 22%. When the required size of the crop is reduced, the crop certainty increases. For in-

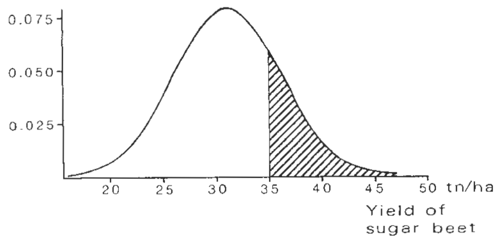


Fig. 5. Calculation of crop certainty (the shaded area/the total area). Yield requirement is 35 000 kg/ha or more.

stance, when the yield should be of 25 000 kg/ha or more, the corresponding crop certainty value is 89%.

Determining the crop certainty by measuring a particular area under the curve is both time-consuming and difficult. It can be calculated much more simply by determining the crop certainty function and then obtaining the crop certainty value from this. Let us denote the probability density function (p.d.f.) of the normal distribution for the size of the 1975 sugar beet yield as $g_S(y_S)$ (Salo) and $g_N(y_N)$ (Naantali). The corresponding crop certainty functions are $W_{gS}(y)$ and $W_{gN}(y)$, where y is the yield requirement. The crop certainty for the 1975 sugar beet yield in the Salo area with a yield requirement of at least y, is obtained as the integral

$$W_{gS}(y) = \int_y^\infty g_S(y_S)\,dy_S. \tag{7}$$

When the normal p.d.f. for the size of the sugar beet yield in the Salo area is taken into consideration, the following equation is obtained:

Fig. 6. The crop certainty function for the Salo (*S*) and Naantali (*N*) areas (eqs. 8 and 9) concerning yield requirements.

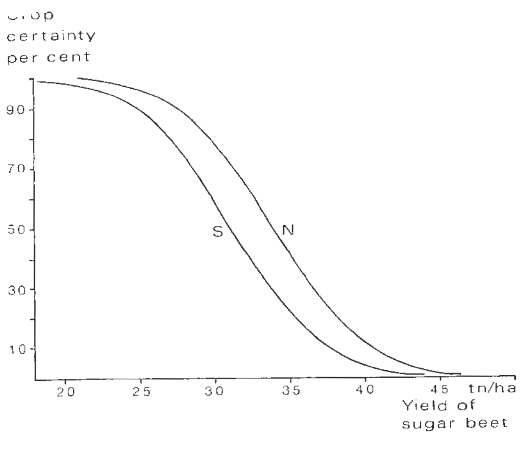


Table 1. *A few yield requirements of sugar beet and corresponding crop certainties in the Salo and Naantali areas*

requirement at least (kg/ha)	Crop certainty (%)	
	Salo	Naantali
25 000	89	96
27 000	79	91
29 000	66	83
31 000	51	71
33 000	36	57
35 000	22	41

$$W_{gS}(y) = \int_y^\infty \frac{1}{\sqrt{2\pi}\cdot 5\,090}\, e^{-[(y_S-31\,100)^2/(2\times 5\,090^2)]}\,dy_S. \tag{8}$$

Similarly, the crop certainty for the 1975 sugar beet yield in the Naantali area with a yield requirement of at least y is obtained as the integral

$$W_{gN}(y) = \int_y^\infty \frac{1}{\sqrt{2\pi}\cdot 5\,120}\, e^{-[(y_N-33\,900)^2/(2\times 5\,120^2)]}\,dy_N. \tag{9}$$

The crop certainty values obtained from eqs. 8 and 9 can be seen in Fig. 6. In addition to the crop certainty corresponding to a certain yield requirement, the size of the yield which at least will be obtained with a certain crop certainty can also be determined (Tables 1 and 2).

Let us denote the normal p.d.f. of the sugar content as $f_S(x_S)$ and $f_N(x_N)$. The corresponding crop certainty functions are $W_{fS}(x)$ and $W_{fN}(x)$, where x is the required sugar content. The crop certainty of the sugar content in the Salo area,

Table 2. *A few crop certainties and corresponding yields of sugar beet in the Salo and Naantali areas*

Crop certainty (%)	Yield at least (kg/ha)	
	Salo	Naantali
90	24 600	27 300
80	26 800	29 600
70	28 400	31 200
50	31 100	33 900
30	33 800	36 600
10	37 600	40 400

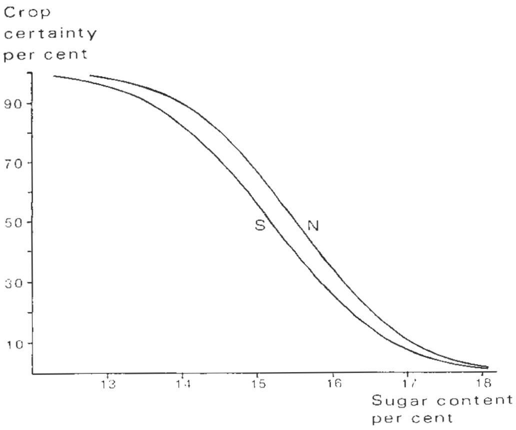


Fig. 7. The crop certainty function for the Salo (*S*) and Naantali (*N*) areas (eqs. 11 and 12) concerning sugar content requirements.

with a quality requirement of at least x, is obtained from the equation

$$W_{fS}(x) = \int_x^{\infty} f_S(x_S)\,dx_S. \tag{10}$$

When the normal p.d.f. (cf. eq. 4) for the sugar content in the Salo area is taken into consideration, the following equation is obtained:

$$W_{fS}(x) = \int_x^{\infty} \frac{1}{\sqrt{2\pi}\cdot 1.25}\, e^{-[(x_S-15.2)^2/(2\times 1.25^2)]}\,dx_S. \tag{11}$$

The corresponding crop certainty for the sugar content of the Naantali area, with a quality requirement of at least x, is obtained from the equation

$$W_{fN}(x) = \int_x^{\infty} \frac{1}{\sqrt{2\pi}\cdot 1.19}\, e^{-[(x_N-15.5)^2/(2\times 1.19^2)]}\,dx_N. \tag{12}$$

Table 3. *A few sugar content requirements of sugar beet and corresponding crop certainties in the Salo and Naantali areas*

Sugar content requirement at least (%)	Crop certainty (%)	
	Salo	Naantali
13.5	91	95
14.0	83	90
14.5	70	80
15.0	55	66
15.5	40	50
16.0	25	34

Table 4. *A few crop certainties and corresponding sugar contents of sugar beet in the Salo and Naantali areas*

Crop certainty (%)	Sugar content at least (%)	
	Salo	Naantali
90	13.6	14.0
80	14.1	14.5
70	14.5	14.9
50	15.2	15.5
30	15.8	16.1
10	16.8	17.0

The crop certainty functions for the sugar content can be seen in Fig. 7. The crop certainties corresponding to certain sugar contents and vice versa are presented in Tables 3 and 4.

Combined Crop Certainty

Combined crop certainty is defined as the probability, calculated from the bivariate normal distribution, when a certain quantity level and simultaneously a certain quality level is required. Let us suppose for example, that the sugar beet yield should be 25 000 kg/ha or more and that the sugar content should be at least 14%. What is the crop certainty of meeting such requirements in the Salo and Naantali areas?

Since the size of the sugar beet yield and the sugar content both have a normal distribution, their joint distribution is a bivariate normal distribution. Let us denote the probability density function of this by $h_S(x_S, y_S)$ and $h_N(x_N, y_N)$. The combined crop certainty $W_S(x, y)$ for the 1975 sugar beet yield in the Salo area, with a yield requirement of at least x, is obtained as the integral

$$W_S(x, y) = \int_x^{\infty}\int_y^{\infty} h_S(x, y)\,dy\,dx. \tag{13}$$

The probability density function of the bivariate normal distribution is

$$h(x, y) = \frac{1}{2\pi\sigma_x\sigma_y\sqrt{1-\varrho^2}}\, e^{-Q(x,y)}, \tag{14}$$

where

$$Q(x, y) = \frac{1}{2(1-\varrho^2)} \times \left\{\frac{(x-\mu_x)^2}{\sigma_x^2} - \frac{2\varrho(x-\mu_x)(y-\mu_y)}{\sigma_x\sigma_y} + \frac{(y-\mu_y)^2}{\sigma_y^2}\right\}. \tag{15}$$

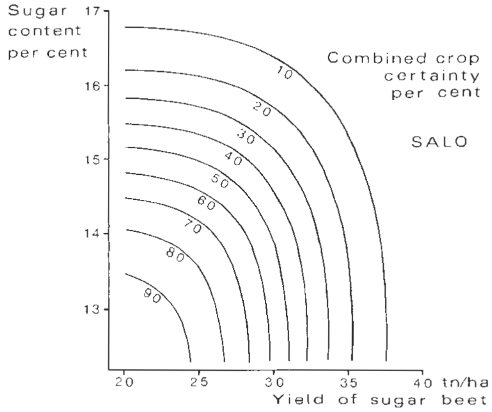


Fig. 8. The combined crop certainty in the Salo area.

In eqs. 14 and 15, σ_x is the standard deviation of the sugar content, σ_y the standard error of the sugar beet yield, μ_x the expected value of the sugar content, and μ_y the trend value of the sugar beet yield. The correlation coefficient (ϱ) for the dependance between the size of the sugar beet yield and the sugar content can be considered to be equal to zero (cf. p. 271) on both the Salo and Naantali areas. Eq. 14 now takes the form

$$h(x, y) = \frac{1}{2\pi\sigma_y\sigma_y} \exp\left(-\frac{(x-\mu_x)^2}{2\sigma_x^2} - \frac{(y-\mu_y)^2}{2\sigma_y^2}\right) \quad (16)$$

or

$$h(x, y) = \left\{\frac{1}{\sqrt{2\pi}\sigma_x} \exp\left(-\frac{(x-\mu_x)^2}{2\sigma_x^2}\right)\right\} \times \left\{\frac{1}{\sqrt{2\pi}\sigma_x} \exp\left(-\frac{(y-\mu_y)^2}{2\sigma_y^2}\right)\right\}. \quad (17)$$

Fig. 9. The combined crop certainty in the Naantali area.
Yield

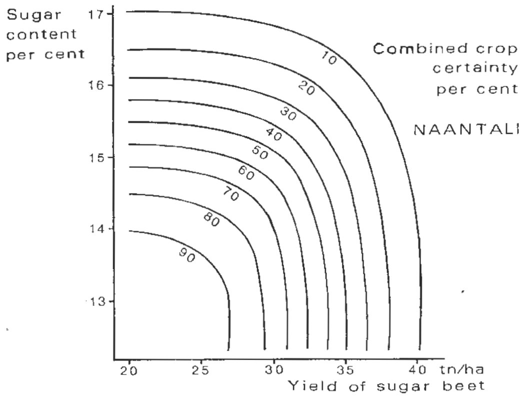


Table 5. *A few yield requirements and simultaneous sugar content requirements of sugar beet and corresponding combined crop certainties in the Salo and Naantali areas*

Yield requirement at least (kg/ha)	Sugar content requirement at least (%)	Combined crop certainty (%)	
		Salo	Naantali
25 000	13.5	80	91
30 000	13.5	53	74
35 000	13.5	20	40
25 000	15.5	35	48
30 000	15.5	23	39
35 000	15.5	8	21

The normal bivariate p.d.f. for the size of the sugar beet yield is thus the product of the p.d.f. of the corresponding yield and the p.d.f. of the sugar content. When this product is combined with eq. 13, then

$$W(x, y) = \int_x^{\infty}\int_y^{\infty} f(x)g(y)\,dy\,dx \quad (18)$$

or

$$W(x, y) = \int_x^{\infty} f(x)\,dx \int_y^{\infty} g(y)\,dy. \quad (19)$$

Thus the combined crop certainty is obtained as the product of the crop certainty of the sugar beet yield and the crop certainty of the sugar content. In the Salo area it is

$$W_S(x, y) = W_{fS}(x)\,W_{gS}(y) \quad (20)$$

and in the Naantali area

$$W_N(x, y) = W_{fN}(x)\,W_{gN}(y). \quad (21)$$

The combined crop certainty for the 1975 sugar beet yield in the Salo and Naantali areas can be determined from the diagrammes presented in Figs. 8 and 9. In addition, a number of crop certainty values are also to be found in Table 5. For example, when a yield of at least 25 000 kg/ha and a quality of at least 14% is required, the combined crop crop certainty for the Salo area is about 73% and for the Naantali area about 86%. The usefulness of Figs. 8 and 9 is further increased by the fact, that the yield axis gives some rough information about the crop certainty for the yield requirements only, and the quality axis about the crop certainty for the mere quality requirements (cf. Figs. 6 and 7).

Discussion

The annual sugar beet yields and the sugar contents, for the areas where sugar beet is cultivated under contract for the sugar extraction factories at Salo and Naantali, were selected as the primary data of this study. Since the whole area under sugar beet is growing under contract and the area in each farm and its annual yield is known, the data is much more accurate than that for other Finnish crops. The sugar content data is also accurate because it is based on yield samples taken from each farm or it has been calculated from the amount of sugar produced from the beet pulp and from the amount of sugar beet processed. A slightly higher sugar beet yield has been obtained in the Naantali area than in the Salo area. The difference is showing an increasing trend. This is due to the fact that sugar beet cultivation is spreading to the Ahvenanmaa island and to the archipelago where the climatic conditions favour this crop. This is also the reason for the slightly higher sugar content in the Naantali area.

The sugar beet yield in both the Salo and Naantali areas showed an increasing trend. This is the result of the developments in sugar beet cultivation which have taken place during the last twenty years. The cultivation methods have been mechanised more and more. The average area per farm under sugar beet cultivation has increased. Similarly, a certain degree of specialisation has taken place with the result that only skilled farmers have continued to grow sugar beet. More productive varieties have also been introduced.

There was no increasing trend in the sugar content. Presumably the sugar content cannot be increased to any great extent by plant breeding. However, it has been found that present varieties have a slightly higher sugar content than those cultivated earlier. The slight increase in the sugar content is masked by the effect of new cultivation methods. Mechanised harvesting has made topping less accurate with the result that there is a decrease in the sugar content of the sugar beet transported to the factory from the farm. This trend will continue. Suggestions have already been made that topping should be discontinued completely.

Both the size of the annual sugar beet yields and the sugar contents have a normal distribution. The causal connection with the normal distribution of the annual weather conditions is apparent. Since for the time being, the annual weather conditions cannot be accurately predicted, they can only be considered from the point of view of probability. Therefore, estimations based on the normal distribution can also be presented about the size of the sugar beet yield and the sugar content.

It seems only natural that the sugar beet yield per hectare and the sugar content should be dependent on each other. For instance, during an unusually warm summer both a heavy yield and a high sugar content would be expected. Conversely, during a cold summer they would both be low. This, however, has not been found the case. The sugar content is determined almost solely by the weather conditions prevailing at the end of the growing season when any considerable increase in the size of the yield can no longer be expected. The sugar content can even be dependant on the light intensity during a couple of days prior to the lifting of the crop. The weather conditions prevailing during the autumn days thus cover the causal relations between the sugar beet yield and the main part of the growing season.

Crop certainty has been determined as a percentage calculated from the normal distribution. Thus the whole concept of crop certainty is quantified and can easily be computed, at least with a computer. Crop certainty can be considered to indicate in how many years out of a hundred will the size or quality level of the yield exceed some particular value. However, as regards the size of the yield this is not exactly correct because the increasing trend in the yield level must be simultaneously taken into account. In addition, it should also be noted that crop certainty only gives the probability that a certain level will be exceeded. Crop certainty does not give the probability, for instance, that a yield of exactly 35 000 kg/ha will be obtained. The probability of this occurrence approaches zero.

The crop certainty values for sugar beet in the Naantali area are all higher than those for the Salo area. This is obviously due to the differences between the primary data of Figs. 2 and 3. When the crop certainty requirements are low, the values for the sugar contents in the Salo and Naantali areas are almost the same (Table 4). This is due to the standard deviations of the sugar contents. A higher value for the standard deviation was obtained for the Salo area. In actual fact, a high sugar content is likely to occur just as frequently in the Salo and Naantali areas—or better just as rarely.

Combined crop certainty combines the certainty of obtaining a certain yield size and quality in one single number. Since it was possible to show that the sugar beet yields and sugar contents were not dependant on each other, a simple calculation method was derived for the combined crop certainty. The combined crop certainty is the product of different crop certainties of the crop size and quality. Thus it is rather small in the case of moderate size and quality requirements. However, the differences between the Salo and Naantali areas are rather clear (Table 5).

In this study determination of the crop certainty is a quick way of obtaining values for the differences in the possibility of achieving yields of required quality and size in different parts of the country. The next step is to study crop certainty on a local basis and to determine its dependence on the geographical location in Finland where the cultivation is carried out.

Summary

In order to determine the possibility of obtaining a yield, the quantity or quality of which fulfills certain requirements, the concept crop certainty was introduced. It was defined as probability calculated from the normal distribution. The yields of sugar beet cultivated under contract for two sugar extraction factories, Salo and Naantali in south-western Finland were studied. Crop certainty functions for both of them were determined. It was seen that when yield requirement was for example 31 000 kg/ha the crop certainties were 51 % and 71 % for the Salo and Naantali areas respectively. When sugar content of for example 15.0 % was required the crop certainties were 55 % (Salo) and 66 % (Naantali). The differences in crop certainties between the two cultivation areas were discussed.

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